

# Structure-stability diagrams and stability-reactivity landscapes: a conceptual DFT study

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**Abstract** Electrophilicity and hardness have been shown to be adequate in constructing structure-stability diagrams. Maximum hardness principle and minimum electrophilicity principle provide a rough guide toward locating the domains of stability and reactivity in a fitness landscape. Bonding in solids, aromaticity, magic alkali clusters, bond—stretch isomers, multivalent superatoms, etc. have been analyzed within this purview.

**Keywords** Electrophilicity · Hardness · Structure · Stability · Reactivity

## 1 Introduction

Conceptual density functional theory [1–3] has been quite successful in providing quantitative definitions of popular qualitative chemical concepts like electronegativity ( $\chi$ ) [4–6], hardness ( $\eta$ ) [7, 8], and electrophilicity ( $\omega$ ) [9–13]. It has also been shown to be adequate in obtaining theoretical justification of the associated electronic structure principles like the electronegativity equalization principle (EEP) [14–16], maximum hardness principle (MHP)

[17–22], hard–soft acid–base principle (HSABP) [23–29], minimum electrophilicity principle (MEP) [30, 31], etc.

Attempts have been made [32] to propose simple structural coordinates to identify and to analyze the stable crystal structures of various solids. Different orbital radii [5, 33, 34], average principle quantum number [35], electronegativity difference [35, 36], hardness sum [36], etc. have been used as coordinates of the related structure-stability diagrams. In the present work, a new set of coordinates in terms of the hardness and the electrophilicity has been provided. An attempt has been made to provide a rationale through the connection between the electronic structure and the geometrical structure encompassing atoms, molecules, and solids. Different electronic structure principles may be used in analyzing the associated reactivity and stability.

Theoretical background of the current work has been presented in Sect. 2. Section 3 provides the computational details, and the results have been discussed in Sect. 4. Finally, Sect. 5 contains some concluding remarks.

## 2 Theoretical background

Global reactivity descriptors like electronegativity [4–6] and hardness [7, 8] are used to describe the reactivity of a molecule as a whole. For an  $N$ -electron system with total energy  $E$ , they are defined [4–8] as the following energy derivatives, within a conceptual density functional theory framework:

$$\chi = -\mu = -\left(\frac{\partial E}{\partial N}\right)_{v(r)} \quad (1)$$

and

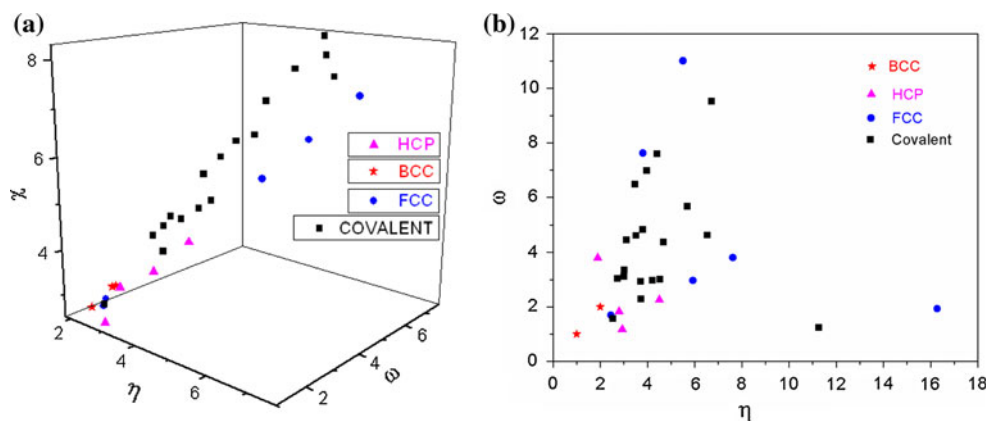
$$\eta = \left(\frac{\partial^2 E}{\partial N^2}\right)_{v(r)} \quad (2)$$

Dedicated to Professor Eluvathingal Jemmis and published as part of the special collection of articles celebrating his 60th birthday.

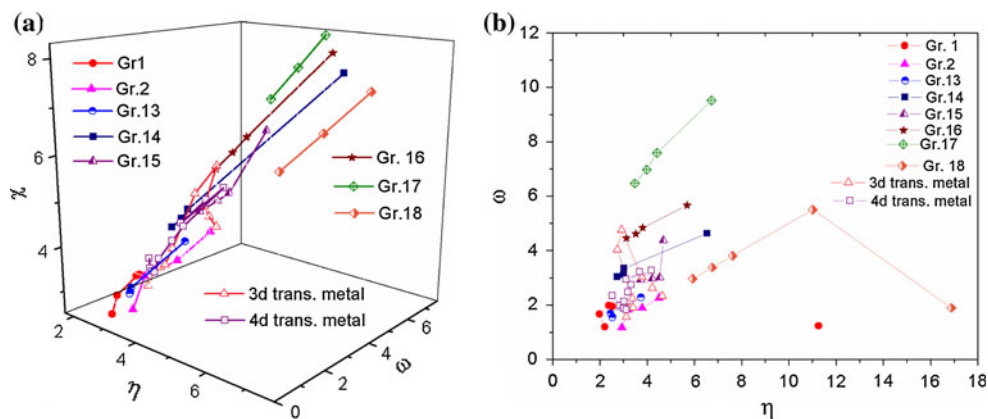
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**Fig. 1** **a** 3-dimensional ( $\chi$ ,  $\eta$ , and  $\omega$ ) and **b** 2-dimensional ( $\omega$  and  $\eta$ ) plots of the crystal structure of the non-transitional elements



**Fig. 2** **a** 3-dimensional ( $\chi$ ,  $\eta$ , and  $\omega$ ) and **b** 2-dimensional ( $\omega$  and  $\eta$ ) plots of the elements up to  $z = 54$



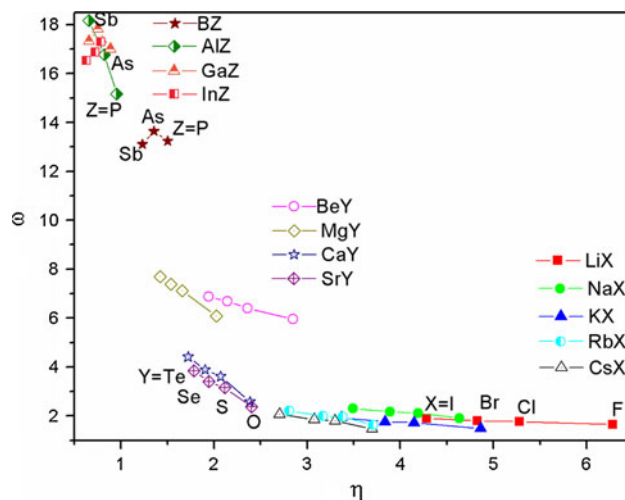
In the above equations,  $\mu$  and  $v(\mathbf{r})$  represent the chemical and external potentials, respectively.

Parr et al. [9] have defined an electrophilicity index ( $\omega$ ) as follows:

$$\omega = \frac{\mu^2}{2\eta} = \frac{\chi^2}{2\eta} \quad (3)$$

For a stable state of a system or a favorable direction of a physico-chemical process, hardness often gets maximized [17–22] and electrophilicity gets minimized [30, 31]. While the maximum hardness principle (MHP) [17–22] is strictly valid [14–16] under the conditions of constant chemical and external potentials [17–22], validity of the minimum electrophilicity principle (MEP) [30, 31] requires that both hardness and electronegativity should attain their maximum values [31].

As hardness and electrophilicity are two important cardinal indices of stability vis-a-vis the two electronic structure principles stated above, it becomes quite legitimate to consider  $\omega$  and  $\eta$  as coordinates in the structure-stability diagrams complementing the previous studies [36] along these lines especially in connection with the nature of bonding in solids. The main aim of this paper is not to generate numbers but to analyze qualitative trends. The present study, in comparison to [36], increases the domain



**Fig. 3**  $\omega$  versus  $\eta$  plot of binary octet compounds showing periodic trends

of applicability encompassing the stability of atoms, molecules, isomers, superatoms, clusters, and solids presumably by strengthening the rationale through the use of two well-accepted electronic structure principles, viz., the principles of maximum hardness and minimum electrophilicity within a broad conceptual DFT framework.

**Table 1** Crystal structure, electronegativity ( $\chi$ , eV), hardness ( $\eta$ , eV), and electrophilicity ( $\omega$ , eV) of binary octet compounds

Compound	Crystal structures	$\chi$	$\eta$	$\omega$	Compound	Crystal structures	$\chi$	$\eta$	$\omega$
LiF	Rock Salt	4.557	6.282	1.653	SrS	Rock Salt	3.652	2.116	3.152
LiCl	Rock Salt	4.323	5.280	1.770	SrSe	Rock Salt	3.636	1.943	3.403
LiBr	Rock Salt	4.175	4.827	1.805	SrTe	Rock Salt	3.703	1.783	3.846
LiI	Rock Salt	4.037	4.283	1.903	ZnO	Wurtzite	5.799	2.358	7.130
NaF	Rock Salt	4.210	4.633	1.912	ZnS	Zinc Blende	5.434	1.808	8.163
NaCl	Rock Salt	4.211	4.191	2.116	ZnSe	Zinc Blende	5.279	1.637	8.510
NaBr	Rock Salt	4.105	3.892	2.165	ZnTe	Zinc Blende	5.115	1.483	8.820
NaI	Rock Salt	4.011	3.495	2.302	BN	Wurtzite	7.598	1.921	15.021
KF	Rock Salt	3.809	4.863	1.491	BP	Zinc Blende	6.312	1.505	13.238
KCl	Rock Salt	3.783	4.149	1.725	BAs	Zinc Blende	6.083	1.356	13.646
KBr	Rock Salt	3.679	3.836	1.764	BSb	Zinc Blende	5.686	1.233	13.110
KI	Rock Salt	3.608	3.371	1.931	AlN	Wurtzite	5.798	1.564	10.751
CsF	CsCl	3.305	3.697	1.477	AlP	Zinc Blende	5.385	0.955	15.176
CsCl	CsCl	3.447	3.299	1.800	AlAs	Zinc Blende	5.254	0.823	16.778
CsBr	CsCl	3.388	3.077	1.865	AlAb	Zinc Blende	4.885	0.657	18.164
CsI	CsCl	3.338	2.704	2.060	GaN	Wurtzite	5.959	1.511	11.752
RbF	Rock Salt	3.494	3.710	1.645	GaP	Zinc Blende	5.491	0.886	17.009
RbCl	Rock Salt	3.634	3.380	1.954	GbAs	Zinc Blende	5.180	0.753	17.817
RbBr	Rock Salt	3.568	3.169	2.009	GaSb	Zinc Blende	4.779	0.659	17.330
RbI	Rock Salt	3.521	2.807	2.208	InN	Wurtzite	5.463	1.431	10.425
BeO	Wurtzite	5.823	2.850	5.949	InP	Zinc Blende	5.223	0.789	17.298
BeS	Zinc Blende	5.497	2.359	6.405	InAs	Zinc Blende	4.957	0.728	16.867
BeSe	Zinc Blende	5.354	2.144	6.686	InSb	Zinc Blende	4.571	0.632	16.522
BeTe	Zinc Blende	5.170	1.943	6.877					
MgO	Rock Salt	4.963	2.025	6.082					
MgS	Rock Salt	4.856	1.656	7.119					
MgSe	Rock Salt	4.760	1.536	7.378					
MgTe	Rock Salt	4.672	1.420	7.684					
CaO	Rock Salt	3.515	2.386	2.590					
CaS	Rock Salt	3.867	2.073	3.606					
CaSe	Rock Salt	3.849	1.905	3.888					
CaTe	Rock Salt	3.899	1.723	4.411					
SrO	Rock Salt	3.377	2.404	2.372					

Moreover, it provides comparable or better results than in [36], in all cases.

### 3 Computational details

In the present study, we analyze the periodic behavior of various atomic systems through the reactivity descriptors. We have taken the calculated values of  $\chi$  and  $\eta$  from Goycoolea et al. [37] except for H, He and Rb as was done before [38], since H and He are not available in Ref. [37] and presumably the corresponding values of Rb were in error. In the case of Rb, we have taken the calculated values of these two parameters from Robles and Bartolotti [39]. Geometry optimization and the frequency calculations

of all the monocyclic hydrocarbons are performed at the B3LYP/6-311+G(d) level of theory by using the Gaussian 03 program [40], and for all the cases, the number of imaginary frequency turns out to be zero which confirms that they correspond to the minima on the potential energy surface (PES). In case of the molecules containing heavy atoms, the geometry optimization as well as frequency calculations are carried out at the B3LYP/6-311+G(d) level of theory for light atoms, whereas B3LYP/SDD level of theory is used for the heavy atoms within the molecules.

We have calculated the ionization potential ( $I$ ) and the electron affinity ( $A$ ) with the help of Koopmans' theorem [41] in terms of the appropriate frontier orbital energies:

$$I = -\varepsilon_{\text{HOMO}}; A = -\varepsilon_{\text{LUMO}} \quad (4)$$

where  $\varepsilon_{\text{HOMO}}$  and  $\varepsilon_{\text{LUMO}}$  are the energies of the highest occupied and lowest unoccupied molecular orbitals, respectively. The global reactivity parameters, electronegativity ( $\chi$ ) and hardness ( $\eta$ ), are then calculated by using the following expressions:

$$\chi = -\mu = -\left(\frac{\partial E}{\partial N}\right)_{v(r)} \approx \left(\frac{I + A}{2}\right) \approx -\left(\frac{\varepsilon_{\text{HOMO}} + \varepsilon_{\text{LUMO}}}{2}\right) \quad (5)$$

and

$$\eta = \left(\frac{\partial^2 E}{\partial N^2}\right)_{v(r)} = \left(\frac{\partial \mu}{\partial N}\right)_{v(r)} \approx I - A \approx (\varepsilon_{\text{LUMO}} - \varepsilon_{\text{HOMO}}) \quad (6)$$

The electrophilicity index ( $\omega$ ) has been calculated with the help of Eq. 3 and the above two equations.

## 4 Results and discussion

Fitness landscapes are constructed for various systems using electrophilicity and hardness as coordinates.

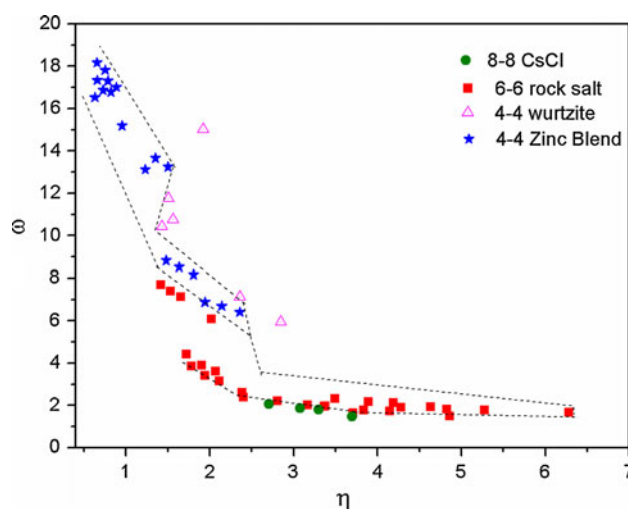
### 4.1 Solids

#### 4.1.1 Bonding in solids and periodicity

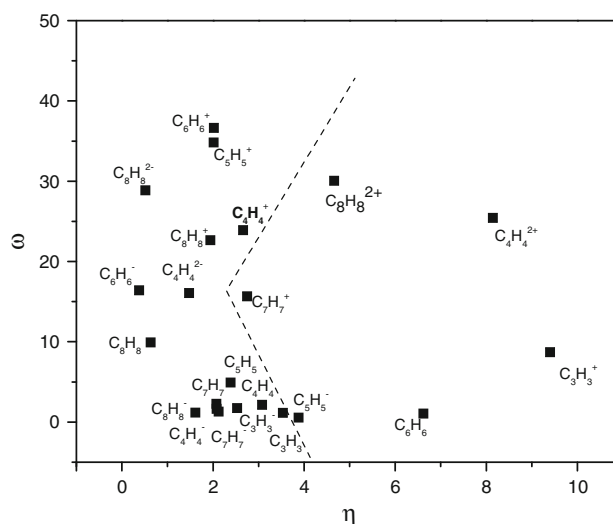
Figure 1a depicts a three-dimensional plot using three reactivity descriptors  $\chi$ ,  $\eta$ , and  $\omega$  as structural parameters for 34 non-transitional elements originally considered in references [34] and [36] (but for H, He, F, and Ne in order that other points are distinctly visible). Conceptual DFT was used in describing solids earlier [42–44]. The most stable metallic structures of these elements as body-centered cubic (bcc), face-centered cubic (fcc), hexagonal closed packed (hcp), and covalent [36] are separated into different domains. In Fig. 1b, the corresponding  $\omega$  versus  $\eta$  plot is shown.

Periodic behavior of these elements are clearly delineated in Fig. 2 (Fig. 2b shows  $\omega$  vs.  $\eta$  plot) in which elements up to  $z = 54$  are included (but for H, He, F, and Ne in order that other points are distinctly visible).

We plot in Fig. 3  $\omega$  versus  $\eta$  for 56 binary octet compounds ( $A^N B^{8-N}$ ) formed from the cations belonging to group-A (Table 1) as was studied in references [34] and [36]. Different zones corresponding to 8–8 CsCl, 6–6 Rock Salt, 4–4 Wurtzite, and 4–4 Zinc Blende structures are discernible. Associated periodic trends are also clearly manifested (Fig. 4). Relative hardness and electrophilicity



**Fig. 4**  $\omega$  versus  $\eta$  plot of binary octet compounds of group-A cations



**Fig. 5**  $\omega$  versus  $\eta$  plot of monocyclic hydrocarbons

values for various groups are as expected from chemical intuition.

### 4.2 Atoms

#### 4.2.1 Stability and electronic structure principles

Stability-reactivity landscapes may be generated by plotting electrophilicity versus hardness. A distinct stability pole would be the higher  $\eta$ -lower  $\omega$  region and a reactivity pole would be lower  $\eta$ -higher  $\omega$  region. Additional stability zones may be conceived via MHP [17–22] (only high  $\eta$ ) or MEP [30, 31] (only low  $\omega$ ). In Fig. 2, the noble gas elements appear in the most stable region, whereas halogens are shown to be highly reactive.

**Table 2** Energy ( $E$ , au), electronegativity ( $\chi$ , eV), hardness ( $\eta$ , eV), and electrophilicity ( $\omega$ , eV) of monocyclic hydrocarbons

Hydrocarbon	$E$	$\chi$	$\eta$	$\omega$
$C_3H_3$	-115.97920	2.834	3.534	1.136
$C_3H_3^-$	-115.95603	-2.970	2.530	1.743
$C_3H_3^+$	-115.75422	12.795	9.405	8.703
$C_4H_4$	-154.70362	3.643	3.077	2.156
$C_4H_4^-$	-154.71283	-2.631	2.082	1.663
$C_4H_4^+$	-154.43119	11.281	2.661	23.911
$C_4H_4^{2-}$	-154.53046	-6.893	1.478	16.077
$C_4H_4^{2+}$	-153.89256	20.352	8.144	25.430
$C_5H_5$	-193.50640	4.854	2.388	4.933
$C_5H_5^-$	-193.56993	-2.093	3.880	0.565
$C_5H_5^+$	-193.18884	11.842	2.013	34.831
$C_6H_6$	-232.30073	3.743	6.617	1.058
$C_6H_6^-$	-232.24588	-3.530	0.380	16.403
$C_6H_6^+$	-231.96608	12.166	2.020	36.626
$C_7H_7$	-270.94913	3.060	2.073	2.258
$C_7H_7^-$	-270.95602	-2.359	2.123	1.311
$C_7H_7^+$	-270.60691	9.285	2.749	15.676
$C_8H_8$	-309.55802	3.535	0.629	9.928
$C_8H_8^-$	-309.56857	-1.942	1.611	1.170
$C_8H_8^+$	-309.33059	9.374	1.940	22.647
$C_8H_8^{2-}$	-309.42749	-5.449	0.514	28.878
$C_8H_8^{2+}$	-308.91526	16.737	4.661	30.052

### 4.3 (Anti) aromatic molecules

#### 4.3.1 Aromaticity

Figure 5 presents the  $\omega$  versus  $\eta$  plot (Table 2) for some monocyclic hydrocarbons and their ions. Majority of the aromatic species [45] with  $(4n + 2)$   $\pi$  electrons fall on the stability domain. Anions and dianions exhibit large  $\omega$  values mainly because of the negative (large)  $\chi$  values and the quadratic appearance of the latter in Eq. 3. It may be

noted that these fitness landscapes provide only a rough guide toward the stability and reactivity and the associated electronic structure principles (MEP and MHP) provide an approximate rationale for the same.

### 4.4 Metal clusters

#### 4.4.1 Magic alkali clusters

It is known that the alkali metal clusters with a magic number of atoms are more stable in comparison to their neighbors [46–50]. We have taken the calculated  $\eta$  and  $\omega$  values for  $Li_n$  and  $Na_m$  clusters from Chattaraj et al. [50]. Figure 6 provides the  $\omega - \eta$  landscapes of  $Li_n$  (Fig. 6a) and  $Na_m$  (Fig. 6b) clusters. As expected, the magic clusters occupy the most stable zone.

### 4.5 Isomers

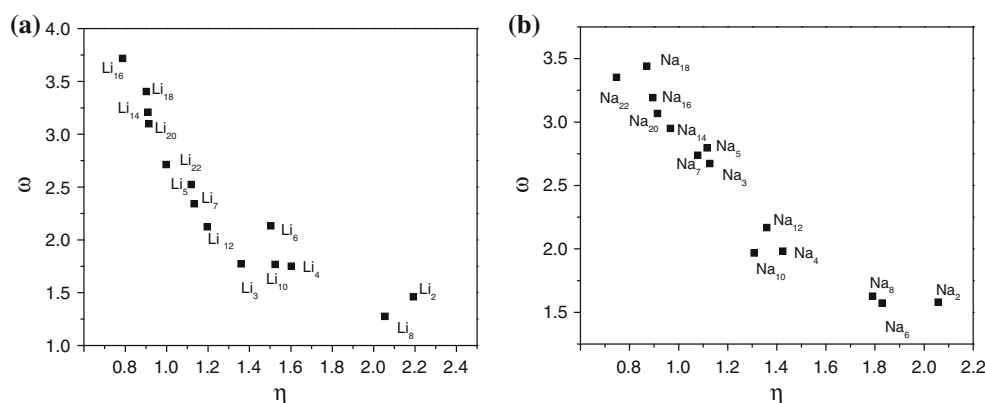
#### 4.5.1 Bond-stretch isomerism

A family of  $Be_6Zn_2^{2-}$  clusters exhibits bond-stretch isomerism [51]. Figure 7 presents the  $\omega - \eta$  landscape of these clusters (shown at the bottom of the Fig. 7). While more stable clusters (according to their relative energy values) appear in the high  $\eta$ -low  $\omega$  region, the three least stable isomers appear in the low  $\eta$ -high  $\omega$  zone. Values of  $\eta$  and  $\omega$  were calculated using a  $\Delta$ SCF method in [51], whereas the present work makes use of Koopmans' theorem.

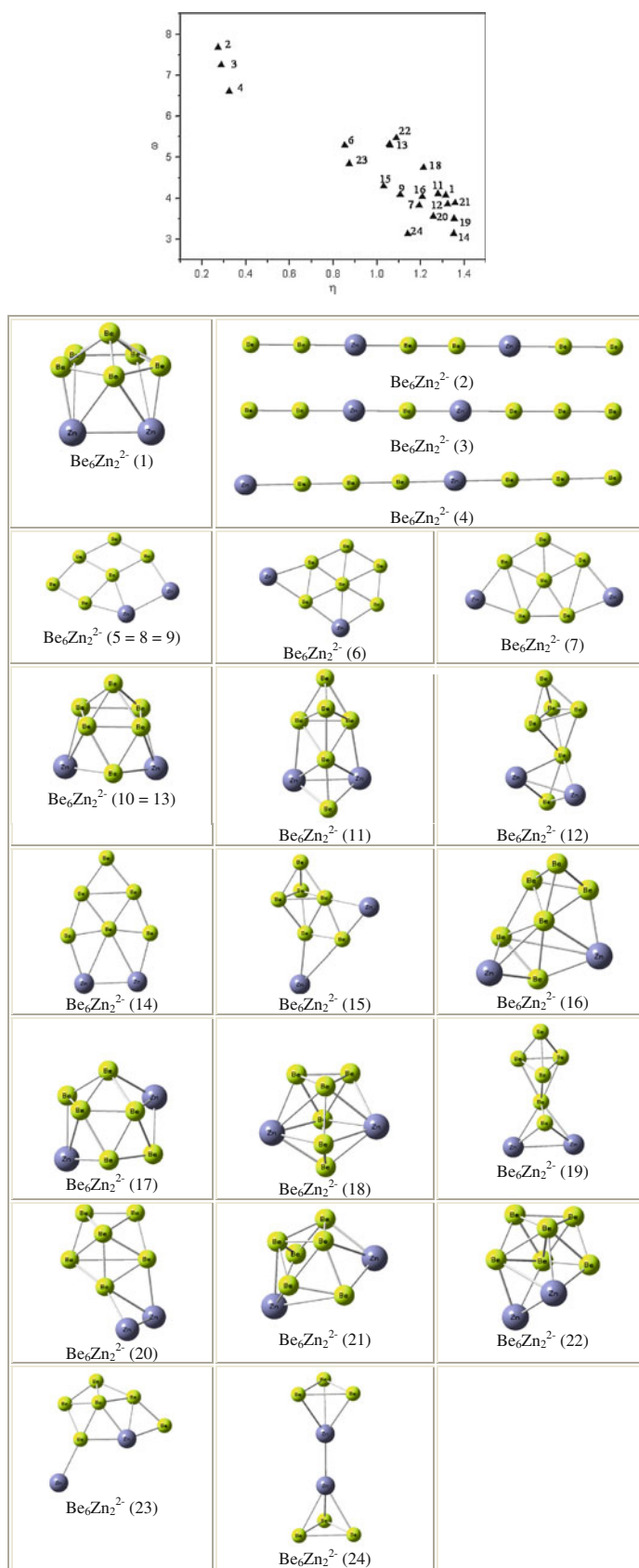
### 4.6 Superatoms

#### 4.6.1 Multivalent superatoms

Some atomic clusters exhibit the characteristics of a single atom in its elemental state [52]. Properties of a halogen atom are revealed by  $Al_{13}$  and those of an alkaline earth metal atom is followed by  $Al_{13}^+$ . Behavior of a noble gas atom is

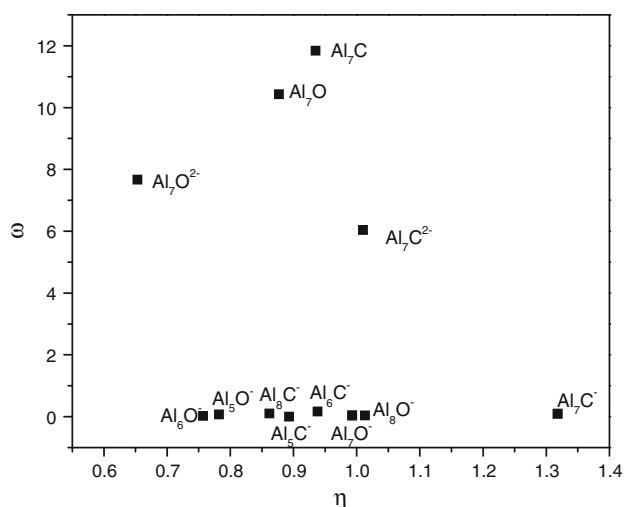
**Fig. 6**  $\omega$  versus  $\eta$  plots of **a**  $Li_n$  and **b**  $Na_m$  clusters

**Fig. 7**  $\omega$  versus  $\eta$  plot of different bond-stretch isomers of  $\text{Be}_6\text{Zn}_2^{2-}$  clusters



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**Fig. 8**  $\omega$  versus  $\eta$  plots of different  $\text{Al}_7\text{M}^{n-}$  ( $\text{M} = \text{C}, \text{O}$ ) clusters and different  $\text{Al}_m\text{M}^{n-}$  ( $\text{M} = \text{C}, \text{O}$ ) clusters ( $m = 5-8$ ;  $n = 0, 2$ )

exhibited by a superatom  $\text{Al}_{13}^-$ . Multivalent superatoms like  $\text{Al}_7\text{C}^-$  and  $\text{Al}_7\text{O}^-$  are aromatic [53] and stable relative to their neighbors. Figure 8 depicts the  $\omega - \eta$  landscape of these superatom ( $\text{Al}_7\text{C}^-$  and  $\text{Al}_7\text{O}^-$ ) and their neighbors formed by varying the number of Al atoms and the number of electrons. The  $\eta$  and  $\omega$  values are taken from Ref. [53]. The stable superatoms occupy the most stable zone. Based on the HOMO–LUMO gaps and the  $\Delta\Delta E$  values  $\text{Al}_7\text{C}^-$  is found to be the most stable cluster among these superatoms, which is faithfully mimicked by the stability-reactivity landscape.

## 5 Concluding remarks

Nature of bonding in different solids can be described by global reactivity descriptors like electrophilicity and hardness. Conceptual density functional theory based electronic structure principles such as the principles of maximum hardness and minimum electrophilicity may be utilized in understanding the stability associated with aromatic systems, magic clusters, bond-stretch isomers, and superatoms through identification of a stability zone in a fitness landscape.

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